Costly Price Discrimination

Peter T. Leeson and Russell S. Sobel
Department of Economics, West Virginia University

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Abstract

In standard microeconomic theory, perfect price discrimination is socially efficient. However, this theory neglects that enacting price discrimination is costly to firms. We prove that when this costliness is accounted for, perfect price discrimination is often socially inefficient. Under linear demand and marginal cost functions, for pure monopolists, perfect price discrimination is sometimes socially inefficient. For monopolistic competitors it is always socially inefficient. Under both market structures, when perfect price discrimination is socially inefficient, it is more inefficient than uniform pricing. Our model is completely general. It applies to all price-searcher market structures, with and without fixed entry costs.
1 Introduction

Is perfect price discrimination socially efficient? A survey of economists, or glance through any economics textbook, would universally suggest the answer is ‘yes.’ Few readers beginning this paper would disagree. The perfectly price discriminating monopolist resolves the deadweight loss by expanding output, creating a social gain, and transfers consumer surplus to himself, which is socially neutral. The net result is increased social welfare. While this reasoning seems sensible, by the end of this paper you will be convinced that it is often wrong.

The ‘obviousness’ of perfect price discrimination’s social efficiency has precluded examinations of its accuracy.\textsuperscript{1} The first, and only, serious challenge to this conventional wisdom is Bhaskar and To (2004) who prove that under monopolistic competition with fixed entry costs, perfect price discrimination can be socially inefficient. Because competitive pressures in these markets fully dissipate profits and consumer surplus is fully extracted, there is no social surplus generated to offset the fixed entry costs to society.\textsuperscript{2}

Building on their insight, we prove that perfect price discrimination is socially inefficient in a much broader class of cases. We show that under linear demand and marginal cost functions, for pure monopolists, perfect price discrimination is sometimes socially inefficient. For monopolistic competitors it is always socially inefficient. Our theory is completely general. In our model perfect price discrimination requires neither monopolistic competition nor fixed entry costs to be welfare reducing. Even under pure monopoly with zero fixed entry costs, perfect price discrimination can be socially inefficient.


\textsuperscript{2}See also, Stole (2005).
We show this occurs because firms face costs of enacting price discrimination.\textsuperscript{3} These costs, which include segmenting consumers, identifying elasticities, and preventing resale, are significant in all industries.\textsuperscript{4} This, of course, is the reason not all firms enact this pricing strategy. The omission of these transactions costs from existing theories of price discrimination is important because, as Varian has pointed out: “A full welfare analysis of attempts to engage in [perfect] price discrimination cannot neglect the transactions costs involved in the negotiation itself” (1989: 604).

The intuition behind our model of how costly perfect price discrimination can lead to inefficiency is straightforward. To enact this pricing strategy a firm would be willing to expend resources up to the amount of its gain from doing so: the sum of the deadweight loss and consumer surplus, both of which are transferred to the monopolist under this price regime. However, the social gain of such a move is only the value of solving the deadweight loss. The private benefit of perfect price discrimination therefore exceeds the social benefit. Although this divergence is not a problem when, as previous theory has assumed, the cost of price discriminating is zero, it becomes a problem if price discrimination is costly. Expenditures to enact price discrimination beyond the value of the deadweight loss are socially wasteful but privately profitable. The firm is expending scarce resources to secure a transfer of consumer surplus that yields no social gain.

Departing from the ubiquitous assumption that price discrimination is costless to the firm, we derive the general and specific conditions under which perfect price discrimination is socially inefficient in the presence of these costs. Section 2 establishes generally (for all price searchers) when enacting perfect price discrimination is

\textsuperscript{3}While the costliness of price discrimination has been recognized in passing by some (see for instance Tirole 2001 and Posner 1975), no one has reconsidered the theory of price discrimination in light of these costs.

\textsuperscript{4}In fact, there is an entire industry known as “yield management” that charges firms for specialized consulting and software to help them implement price discrimination. These services can be extremely costly, with the software alone costing upwards of $10 million per year for a single large firm (see Lieberman 1991, 1993).
privately profitable but socially inefficient. Section 3 does this specifically for the case of pure monopoly, while Section 4 does so for the case of monopolistic competition. Section 5 concludes.

2 Costly Perfect Price Discrimination and Social Efficiency: The General Case

Figure 1 depicts the traditional welfare analysis for a single-price price searcher. The area of triangle \( cde \) is the deadweight loss to society associated with the price searcher’s market power. Consumer surplus is given by the area of triangle \( abc \).

Suppose this producer wants to convert his pricing strategy to one of perfect price discrimination. For reasons discussed above, doing so is costly. Let \( C \) be the price searcher’s cost of enacting perfect price discrimination.\(^5\) For ease of exposition, here we assume the cost of perfectly price discriminating is a fixed cost of adopting the new pricing strategy. However, the results we derive apply equally when there are variable costs of perfectly price discriminating, though the interpretation changes slightly (see the Appendix). We also assume that the cost of implementing perfect price discriminating is industry specific.

To determine the profitability of implementing perfect price discrimination, the price searcher must weigh the benefits of enacting perfect price discrimination against the cost, \( C \). His benefit of perfectly price discriminating consists of two components. The first consists of transforming the area of the deadweight loss triangle \( cde \) into producer surplus. Formally, this area can be written as \( \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \), which represents the lost gains from trade under a single-pricing strategy. In the traditional

\(^5\) This assumes costless uniform pricing. If uniform pricing is also costly, \( C \) refers to the additional cost of adopting the more complex pricing strategy of perfect price discrimination.
perfect price discrimination analysis, this portion of the price searcher’s benefit is generally the only benefit considered.

Equally important to the price searcher, however, is the second component of his benefit from implementing perfect price discrimination. This is the triangle abc. The area of this triangle, \( \int_{0}^{Q_m} [P(Q) - P_m] \), constitutes the consumer surplus enjoyed by consumers under uniform pricing, which is transferred to the price searcher by enacting perfect price discrimination.

This second portion of the price searcher’s benefit from perfectly price discriminating, the transferred consumer surplus, does not contribute to social welfare as does the first portion of his benefit, the transformed deadweight loss. There is no efficiency gain from transferring consumer surplus under a single-price strategy to producer surplus under a strategy of perfect price discrimination; there is only a transfer. Thus, the transferred consumer surplus represents a private benefit that is not a social benefit.

When the price searcher’s total private benefit of perfectly price discriminating, \( \int_{0}^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \) (transferred consumer surplus plus transformed deadweight loss), is greater than his cost of enacting perfect price discrimination, \( C \), he will do so. In contrast, where his total private benefit of this pricing strategy is less than its cost, he will choose not to enact perfect price discrimination.

Two possibilities, with contradictory social welfare effects, arise when the price searcher finds perfect price discrimination privately profitable. In the first case, the cost of perfect price discrimination is less than or equal to the deadweight loss of uniform pricing, \( C \leq \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \). When he implements perfect price discrimination society gains. The deadweight loss of uniform pricing is removed, and it is removed at a cost less than the social benefit of doing so.

In the second case, the cost of enacting perfect price discrimination is higher, and exceeds the deadweight loss of maintaining uniform pricing. Since the price searcher’s
benefit from pursuing perfect price discrimination equals \( Q_m \int_0^{P(Q) - P_m} + \int_{Q_m}^{P(Q) - MC(Q)} \), he is willing to expend resources up to this point to implement perfect price discrimination. Here, however, a problem emerges. While it is privately profitable for the price searcher to pursue perfect price discrimination in this case, it is social welfare reducing. Every dollar spent by the price searcher beyond \( \int_{Q_m}^{P(Q) - MC(Q)} \) to implement perfect price discrimination generates a benefit to him by transforming would-be consumer surplus into producer surplus. But from the perspective of social welfare, these dollars are a waste—resources expended only to affect the transfer of other resources. The effect is like that originally described by Tullock (1967) on the social losses of rent seeking. The price searcher’s private cost/benefit calculus can lead him to pursue price discrimination when it is socially inefficient. This leads us to our first proposition.

**Proposition 1** For all price searchers, perfect price discrimination is privately profitable but socially inefficient when the cost of enacting perfect price discrimination exceeds the deadweight loss generated under uniform pricing, but is less than or equal to the sum of transformed deadweight loss and transferred consumer surplus.

**Proof.** From above, the price searcher’s private benefit from enacting perfect price discrimination is:

\[
Q_m \int_0^{P(Q) - P_m} + \int_{Q_m}^{P(Q) - MC(Q)}
\]

The social benefit from the price searcher enacting perfect price discrimination is only:

\[
\int_{Q_m}^{P(Q) - MC(Q)}
\]

So, when \( \int_0^{P(Q) - P_m} + \int_{Q_m}^{P(Q) - MC(Q)} \geq C > \int_{Q_m}^{P(Q) - MC(Q)} \):

\begin{align*}
(A) & \quad \int_0^{P(Q) - P_m} + \int_{Q_m}^{P(Q) - MC(Q)} - C \geq 0; \text{ enacting perfect price discrimination is profitable for the price searcher, and}
\end{align*}
In the analysis that follows we consider only those cases for which enacting perfect price discrimination is privately profitable. Thus, we assume that $C$ always satisfies (A). Let us call (B) the ‘social inefficiency condition.’ This condition implies an important fact about the relative inefficiency of perfect price discrimination and uniform pricing.

**Proposition 2** When perfect price discrimination is socially inefficient, it is always more socially inefficient than uniform pricing.

**Proof.** From Proposition 1 we know that enacting perfect price discrimination is privately profitable and socially inefficient when:

$$C > 0; \text{enacting perfect price discrimination is socially inefficient.}$$

We also know that the social inefficiency of maintaining uniform pricing is:

$$C \geq 0.$$  

From this, it immediately follows that whenever $C$ satisfies the social inefficiency condition, it is also more socially inefficient than maintaining uniform pricing.

3 Costly Perfect Price Discrimination Under Pure Monopoly

A pure monopoly may be the result of government-erected barriers to entry—a special grant of monopoly privilege, or may emerge “naturally” in the market when there are significant economies of scale in an industry. Pure monopolists, like all price searchers, have a choice about the kind of pricing scheme they will follow, and also like all price searchers, they face a cost of perfectly price discriminating. Because they do not face competition, however, pure monopolists can earn positive economic
profit. There are no forces tending to push this profit to zero. Thus, here we consider only pure monopolists earning positive economic profit.\(^6\)

To understand when enacting perfect price discrimination is socially inefficient, it is useful to first think about the issue intuitively. Consider Figure 2.

[Figure 2 about here.]

MC depicts the situation of constant marginal production costs. Under this marginal cost function, the threshold of socially inefficient perfect price discrimination is given by the triangle \(abc\)—the deadweight loss transformed into producer surplus by enacting perfect price discrimination. Once the cost of enacting perfect price discrimination exceeds this area, it becomes socially inefficient. Now consider \(MC'\), the case in which marginal production costs are increasing. As \(MC\) rotates up to \(MC'\), the deadweight loss eliminated by enacting perfect price discrimination becomes smaller.

To see this, compare the size of the deadweight loss when the price searcher faces constant marginal production cost (\(MC\))—triangle \(abc\) considered before, to the size of the deadweight loss when the price searcher faces rapidly increasing marginal production costs (\(MC'\))—triangle \(abd\). Crossing the inefficiency threshold in this case therefore requires only a relatively small cost of enacting perfect price discrimination.

The reverse happens when marginal production costs are decreasing in output, as they are for \(MC''\). Here, the inefficiency threshold of enacting perfect price discrimination grows substantially larger than under constant marginal cost, \(MC\). The deadweight loss eliminated by perfect price discrimination enlarges to the triangle \(abe\). Enacting perfect price discrimination needs be quite costly before its implementation is socially inefficient. The growing (shrinking) deadweight loss solved by enacting perfect price discrimination as marginal production costs move from steeply increasing (decreasing) to steeply decreasing (increasing) suggests that for a given

\(^6\)A pure monopolist could earn exactly zero economic profit, in which case he formally resembles a monopolistic competitor, which we consider in the subsequent section.
cost of enacting perfect price discrimination, $C$, the likelihood that enacting perfect price discrimination is socially inefficient decreases (increases) as marginal production costs go from steeply increasing (decreasing) to steeply decreasing (increasing). With this in mind we are ready for our first proposition regarding pure monopoly.

**Proposition 3** For a pure monopolist who faces linear demand and marginal production costs, enacting perfect price discrimination is socially inefficient if and only if the slope of marginal production cost is greater than the product of the (absolute value of the) slope of demand and the price searcher’s rate of return on his investment in enacting perfect price discrimination minus one.

**Proof.** Let $C'$ be the share of the potential benefits a price searcher receives by perfectly price discriminating that he must spend to successfully implement perfect price discrimination. So,

$$C' = \frac{C}{\int_0^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)]}$$

Since enacting perfect price discrimination is costly, $C' > 0$. Further, because the pure monopolist we consider earns positive economic profit, $C' < 1$. Thus, $0 < C' < 1$. From the social inefficiency condition we know that enacting perfect price discrimination is socially inefficient if and only if: $C > \int_{Q_m}^{Q_c} [P(Q) - MC(Q)]$. Re-writing this in terms of $C'$ gives us:

$$C' \left( \int_0^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \right) > \int_{Q_m}^{Q_c} [P(Q) - MC(Q)]$$

Therefore, to determine when enacting perfect price discrimination is socially inefficient we need to establish when this inequality holds.

To do this, consider a pure monopolist who faces a linear demand function of the form: $P = \alpha - \beta Q$, where $\alpha, \beta > 0$ and $\beta$ is the (absolute value of the) slope of
demand. His total production cost under uniform pricing is given by the function:

\[ TC = \delta + \sigma Q + \phi Q^2, \]

which results in a linear marginal cost function, where \( \delta, \sigma \geq 0, \phi \) may be positive (in the case linearly increasing marginal production costs), negative (in the case of linearly decreasing marginal production costs), or zero (in the case of constant marginal production costs), and \( 2\phi \) is the slope of marginal production cost. His total cost when he enacts perfect price discrimination is therefore:

\[ TC = \delta + \sigma Q + \phi Q^2 + C, \]

where \( C \) is the fixed cost of perfect price discrimination. The monopolist’s problem is as follows: First, determine his profit-maximizing quantity \( (Q_m) \) and price \( (P_m) \):

\[
\Pi_{\text{max}} = P(Q)Q - TC(Q) = (\alpha - \beta Q)Q - (\delta + \sigma Q + \phi Q^2 + C)
\]

His F.O.C. is thus given by:

\[
\frac{\partial \Pi}{\partial Q} = (\alpha - \sigma) - 2(\beta + \phi)Q = 0
\]

Solving for \( Q_m \):

\[
Q_m = \frac{\alpha - \sigma}{2(\beta + \phi)}
\]

Substituting and solving for \( P_m \) gives:

\[
P_m = \alpha - \beta Q_m = \frac{\alpha \beta + 2\alpha \phi + \beta \sigma}{2(\beta + \phi)}
\]

To calculate \( \int_0^{Q_m} [P(Q) - P_m] \) and \( \int_0^{Q_c} [P(Q) - MC(Q)] \), we need to evaluate MC at \( Q_m \):

\[
MC[Q_m] = \frac{\sigma \beta + \phi \alpha}{\beta + \phi}
\]

We must also calculate the competitive level of output, \( Q_c \). To do this set \( P = MC \):
\[ \alpha - \beta Q = \sigma + 2\phi Q. \] And solving for \( Q_c \):

\[ Q_c = \frac{\alpha - \sigma}{2\phi + \beta} \]

Now, calculate \( \int_0^{Q_m} [P(Q) - P_m] : \)

\[ \int_0^{Q_m} [P(Q) - P_m] = \frac{\beta(\alpha - \sigma)^2}{8(\beta + \phi)^2} \]

And \( \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] : \)

\[ \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] = \frac{\beta^2(\alpha - \sigma)^2}{8(\beta + \phi)^2 (2\phi + \beta)} \]

Now, substitute back into \( C' \left( \int_0^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \right) > \int_{Q_m}^{Q_c} [P(Q) - MC(Q)] \), which yields:

\[ C' \left[ \frac{\beta(\alpha - \sigma)^2}{8(\beta + \phi)^2} + \frac{\beta^2(\alpha - \sigma)^2}{8(\beta + \phi)^2 (2\phi + \beta)} \right] > \frac{\beta^2(\alpha - \sigma)^2}{8(\beta + \phi)^2 (2\phi + \beta)} \]

Solving for \( 2\phi \) (the slope of MC):

\[ 2\phi > 2\beta \left( \frac{1}{2C'} - 1 \right) \]

Note that since \( C' = \frac{C}{\int_0^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)]} \), \( \left( \frac{1-C'}{C} \right) \) is equal to the monopolist’s rate of return on his investment in enacting perfect price discrimination, which we call \( r \). So, this expression may be re-written in terms of the monopolist’s rate of return on investment in enacting perfect price discrimination as:

\[ 2\phi > \beta(r - 1) \]
4 Costly Perfect Price Discrimination Under Monopolistic Competition

The foregoing discussion established under what conditions enacting perfect price discrimination, for a pure monopolist who found this pricing strategy profitable, is socially inefficient. This section establishes the same for monopolistic competitors. The critical difference here is that monopolistic competitors face competitive pressures from their rivals that tend toward the elimination of economic profit in equilibrium. In contrast to pure monopolists, economic profit is equal to zero for these price searchers, even on their price discriminating activities.

This competitive pressure can be thought of in two ways. On the one hand, the competition of monopolistic competitors for consumers in a particular industry tends to drive down the demand that each firm faces in the industry, reducing each firm’s economic profit to zero in equilibrium. Alternatively, the competitive pressure of rival price searchers for resources—including those used to successfully enact perfect price discrimination—tends to drive up the price of these resources such that in equilibrium each monopolistically competitive price searcher is expending resources on perfectly price discriminating and other activities equal to the benefit of these resources in generating him revenue. Economic profit is again driven to zero.

The zero economic profit condition for monopolistic competitors in equilibrium therefore means that for each price searcher in an industry, the cost of implementing perfect price discrimination is equal to the price searcher’s private benefit of doing so: $C = \int_{0}^{Q_m} [P(Q) - P_m] + \int_{Q_m}^{Q_c} [P(Q) - MC(Q)]$. So, under monopolistic competition, $C' = 1$, or equivalently, $r = 0$.

One other important change needed to modify the model from pure monopoly to
monopolistic competition deals with the range of potential marginal production cost functions that price searchers under monopolistic competition may face. Although pure monopolists may face decreasing marginal production costs, monopolistic competitors cannot. If they did, the situation would result in a “natural” monopoly, meaning there is only one price searcher in the industry, as in the case we considered in the previous section. Under monopolistic competition the slope of the marginal production cost curve must be greater than or equal to zero, so $2\phi \geq 0$. With this in mind, consider Proposition 4.

**Proposition 4** For monopolistic competitors who face linear demand and marginal production costs, enacting perfect price discrimination is always socially inefficient.

**Proof.** To prove this proposition we simply follow the procedure for proving Proposition 3. From the social inefficiency condition we know that for perfect price discrimination to be socially inefficient, the cost of enacting it must satisfy: $C > \frac{Q_c}{Q_m} \int \left[ P(Q) - MC(Q) \right]$. Re-written in terms of $C'$ this was:

$$C' \left( \frac{Q_m}{0} \int \left[ P(Q) - P_m \right] + \frac{Q_c}{Q_m} \int \left[ P(Q) - MC(Q) \right] \right) > \frac{Q_c}{Q_m} \int \left[ P(Q) - MC(Q) \right]$$

Using substitution:

$$C'' \left\{ \beta (\alpha - \sigma)^2 + \frac{\beta^2 (\alpha - \sigma)^2}{8(\beta + \phi)^2 + (2\phi + \beta)} \right\} > \frac{\beta^2 (\alpha - \sigma)^2}{8(\beta + \phi)^2 + (2\phi + \beta)}$$

Solving for $2\phi$ (the slope of MC):

$$2\phi > 2\beta \left( \frac{1}{2C''} - 1 \right)$$
Under monopolistic competition, zero economic profit implies $C' = 1$, so this gives:

$$2\phi > -\beta$$

Since under monopolistic competition $2\phi \geq 0$, this is always satisfied. ■

Note that this condition for social inefficiency applies even when $\delta = 0$, implying no fixed cost of market entry for the firm. Thus, unlike previous analyses of monopolistic competition that require fixed entry costs for perfect price discrimination to be inefficient, our theory does not.

5 Conclusion

We have demonstrated how under plausible conditions price searchers are led to pursue “too much” perfect price discrimination, generating welfare losses even when perfect price discrimination is used. This occurs because price discrimination is costly to implement and the firm will invest resources both to transform the deadweight loss, which creates a social benefit, and transfer existing consumer surplus to itself, which does not. For pure monopolists, enacting perfect price discrimination is sometimes socially inefficient. For monopolistic competitors, enacting perfect price discrimination is always socially inefficient. For both market structures, when perfect price discrimination is socially inefficient, it is always more inefficient than if uniform pricing were maintained.

The straightforwardness of our argument, once it has been explained, makes it susceptible to the claim that it is ‘obvious.’ However, the obviousness of our result is rivaled only by the alleged obviousness of perfect price discrimination’s social efficiency. Of course, both cannot simultaneously be true.

The proposition that perfect price discrimination is always socially efficient remained unquestioned until Bhaskar and To (2004). Their important work showed
that under monopolistic competition with fixed entry costs, perfect price discrimination can be socially inefficient. Here we build on their conclusion to provide a general theory of perfect price discrimination’s social inefficiency, which also holds for pure monopolists and in cases with zero fixed entry costs. The widely-held belief that perfect price discrimination is socially efficient is conditional on ignoring all costs of enacting price discrimination. However, these costs are generally substantial and always exist. Once they are incorporated into the analysis, the social efficiency of perfect price discrimination is no longer obvious. While some authors have mentioned the costs of enacting price discrimination in passing, our analysis is the first to model these costs and derive their implications.

We examined the inefficiency of costly perfect price discrimination only under the relatively simple cases of linear demand and linear marginal cost functions. Future analyses should investigate whether the propositions regarding the inefficiency of perfect price discrimination derived in this paper also hold under less simplifying assumptions about demand and production costs.

We also circumscribed our discussion to the case of perfect price discrimination. But there is good reason to think that the general problem of inefficiency we described also applies to second- and third-degree price discrimination. Here too, if price discrimination is costly, since the price searcher’s benefits of price discrimination involve not only the transformed deadweight loss under uniform pricing but also the transferred consumer surplus, in a subset of cases price discrimination will prove privately profitable but be socially inefficient.
References


Steps to Enhanced Revenues,” in Arthur D. Little Inc., Travel, Tourism and
Hospitality Bulletin.


Journal, 66-83.


[16] Spence, Michael (1976). “Product Selection, Fixed Costs, and Monopolistic Com-

nomics, 1-18.

Armstrong and R. Porter eds. Handbook of Industrial Organization Vol. III. Am-
sterdam: North-Holland.

Press.


nomic Review, 870-875.
Appendix

Here we consider the case when the price searcher’s cost of practicing perfect price discrimination includes variable cost components (with respect to the firm’s output), either alone, or in addition to fixed cost components. Let a more generalized form of the cost of perfect price discrimination to the firm, $C$, be:

$$C = c_0 + c_1 Q + c_2 Q^2$$

where $c_0 \geq 0$ is the fixed cost of enacting perfect price discrimination. The variable cost component is determined by $c_1$ and $c_2$. When $c_2 = 0$, and $c_1 > 0$, then $c_1$ measures the constant per unit cost of perfectly price discriminating (a cost per unit of preventing resale, for example). When $c_1 > 0$ and $c_2 > 0$, the marginal cost (per additional unit of output) of perfectly price discriminating is $\frac{\partial C}{\partial Q} = c_1 + 2c_2 Q$, which can be either an increasing (when $c_2 > 0$) or decreasing (when $c_2 < 0$) amount per unit as additional units of output are produced. In the presence of $c_1$ costs alone, the intercept of the firm’s marginal cost curve is increased, resulting in a parallel upward shift in the marginal cost curve. When $c_2$ costs are present, the slope of the firm’s marginal cost curve is also affected. This can be seen more clearly in the firm’s new total cost function:

$$TC = \delta + \sigma Q + \phi Q^2 + C = \delta + \sigma Q + \phi Q^2 + (c_0 + c_1 Q + c_2 Q^2) = (\delta + c_0) + (\sigma + c_1) Q + (\phi + c_2) Q^2$$

which produces a marginal cost function of:

$$MC = (\sigma + c_1) + 2(\phi + c_2) Q$$
These two equations may be re-written using $\delta' = \delta + c_0$, $\sigma' = \sigma + c_1$, and $\phi' = \phi + c_2$ as:

$$TC' = \delta' + \sigma'Q + \phi'Q^2$$

and

$$MC' = \sigma' + 2\phi'Q$$

Thus, including variable costs of perfect price discrimination in our analysis simply results in all appearances of $\sigma$ and $\phi$ being replaced by $\sigma'$ and $\phi'$ (note that $\delta'$ is already used in our analysis because we include both $\delta$, which can be zero or positive, and $C$, our equivalent of the fixed cost of enacting perfect price discrimination $c_0$). Because $\sigma$, like $\delta$, falls out of the equations prior to the final solutions in all of our propositions, constant per unit variable costs of the $c_1$ variety do not change our results.

However, when the variable cost of perfectly price discriminating either increases or decreases with output because of $c_2$ costs, altering the slope of the firm’s marginal cost curve, the interpretation of our propositions involving $\phi$ changes slightly. In Proposition 3, where the social inefficiency condition is $2\phi > \beta(r - 1)$, this now becomes $2\phi' > \beta(r - 1)$, or $2(\phi + c_2) > \beta(r - 1)$. The intuition remains the same in that we are still referring to the slope of the firm’s marginal cost curve ($2\phi$), but this slope is now the slope of marginal cost inclusive of any slope-altering marginal costs related to enacting perfect price discrimination. Proposition 4, where the social inefficiency condition is $2\phi > -\beta$, is similarly affected to become $2(\phi + c_2) > -\beta$. 
Figure 1. The Price Searcher’s Private Benefit from Perfect Price Discrimination
Figure 2. Marginal Cost and the Social Inefficiency of Perfect Price Discrimination